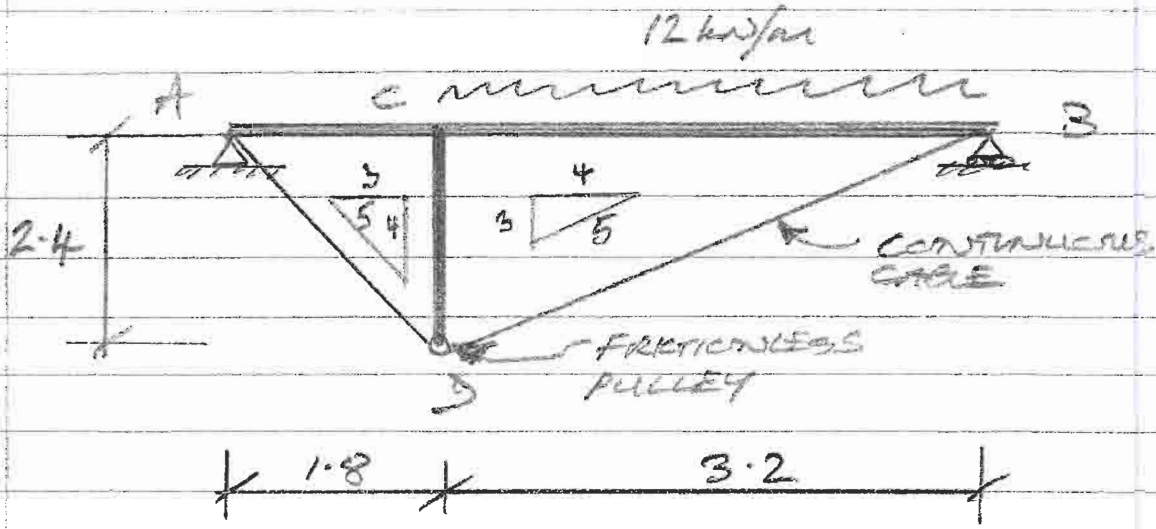
EXAMPLE 1



THE BEAM ACB AND STRUT CD ARE RIGIDLY CONNECTED AT C. CABLE ADB IS CONTINUOUS OVER THE FRICTIONLESS PULLEY AT D.

EI VALUES: $ACB = 2EI = 8 \times 10^3 \, kNm^2$
$CD = EI = 4 \times 10^3 \, kNm^2$

EA VALUES: $ADB = 16 \times 10^3 \, kN$

AXIAL EFFECTS IN AB & CD ARE IGNORED.

THE STRUCTURE IS INDET. TO 1° IE. IF WE REMOVE THE CABLE IT IS A STAT. DET. BEAM.

=> SELECT P_{ADB} (THE CABLE FORCE) AS THE REDUNDANT FORCE.

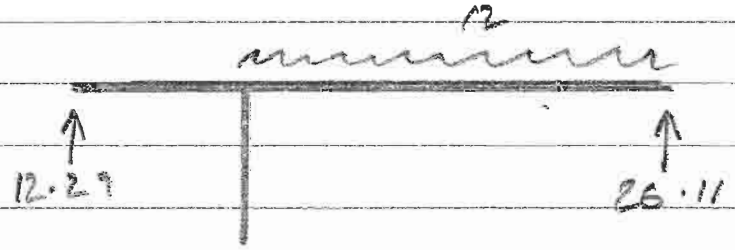
USING VIRTUAL WORK TO SOLVE, WE HAVE:

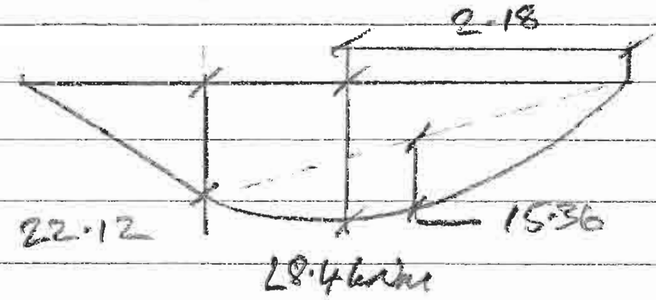
P_1 = VIRTUAL FORCE
M_1 = VIRTUAL MOMENT } DUE TO APPLIED VIRTUAL FORCE (UNIT FORCE)

ρ = REAL AXIAL DISP.
μ = REAL ROTATION

P = REAL FORCE
M = REAL MOMENT } DUE TO REAL APPLIED LOADS

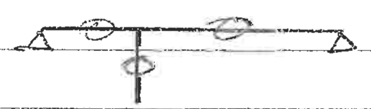
P_0, M_0 ARE REAL FORCES/MOMENTS IN THE CUT-BACK STRUCTURE DUE TO THE REAL APPLIED LOADS:





BMD.

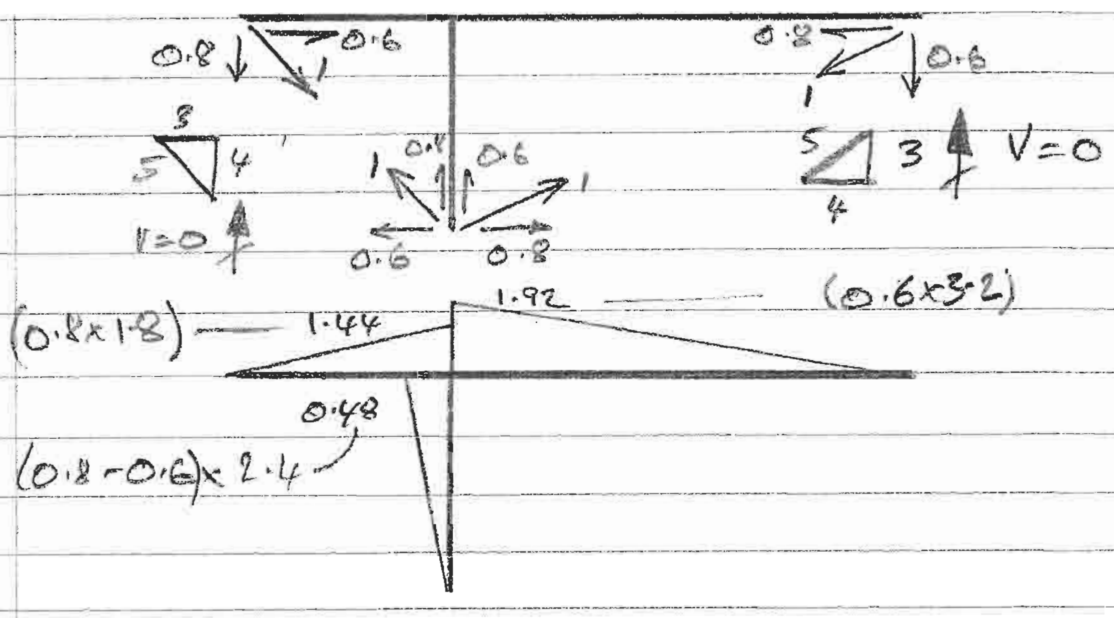
M_0



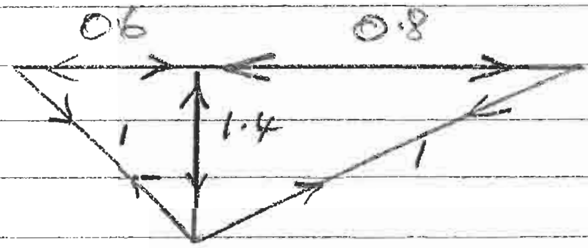
P_0 = 0 IN CASE ALSO.

P_0

APPLY A VIRTUAL FORCE (UNIT LOAD) IN LIEU OF THE REDUNDANT:



M_1



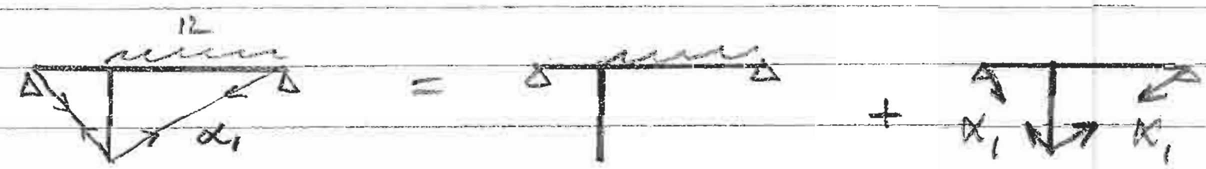
P_1

EXTERNAL VIRTUAL WORK = INTERNAL VIRTUAL WORK.

EX. VIRTUAL FORCE × EX. REAL DISPLACEMENT = INT. VIRTUAL FORCE × INT. REAL FORCE

$$1 \times 0 = \Sigma P_1 \rho + \Sigma M_1 m$$

ρ IS A REAL DISPLACEMENT AND m IS A REAL ROTATION:



$$P = P_0 + \alpha_1 P_1$$
$$M = M_0 + \alpha_1 M_1$$

α_1 IS THE UNKNOWN FORCE
P_1 & M_1 ARE FORCES/MOMENTS FROM A UNIT LOAD.

HENCE A REAL DISPLACEMENT IS:

$$\rho = \frac{PL}{EA} = \frac{(P_0 + \alpha_1 P_1)L}{EA}$$

FOR A <u>SINGLE</u> MEMBER, SO SUM FOR ALL MEMBERS.

SIMILARLY, FROM MOHR I:

$$m = \frac{M\,ds}{EI} = (M_0 + \alpha_1 M_1)\frac{ds}{EI}$$

AGAIN, SUM FOR ALL MEMBERS. RETURNING TO THE V.W EXPRESSION WE NOW SEE

$$0 = \Sigma P_1 (P_0 + \alpha_1 P_1)\frac{L}{EA} + \Sigma M_1 (M_0 + \alpha_1 M_1)\frac{ds}{EI}$$

$$= \Sigma P_1 P_0 + \alpha_1 \Sigma \frac{P_1^2 L}{EA} + \Sigma M_1 M_0 \frac{ds}{EI} + \alpha_1 \Sigma \frac{M_1^2 ds}{EI}$$

NOW WE CALCULATE EACH OF THE TERMS IN THE ABOVE EXPRESSION:

- $\Sigma P_1 P_0$

P_0 / M_0 RELEASED STRUCTURE: $P_0 = 0$

$$\Rightarrow \Sigma P_1 P_0 = 0$$

- $\Sigma \frac{P_1^2 L}{EA}$

AS WE ARE NEGLECTING AXIAL EFFECTS IN THE BEAM MEMBERS ACB & CD, WE ONLY NEED EVALUATE THE EXPRESSION FOR MEMBERS AD, DB.

MEMBER	P_1	P_1^2	L	EA	$P_1^2 L / EA$
AD	1	1	3	16×10^3	1.875×10^{-4}
DB	1	1	4	16×10^3	2.500×10^{-4}
					$\Sigma = 0.438 \times 10^{-3}$

ALTHOUGH WE COULD EASILY EVALUATE TWO MEMBERS AS:

$$\frac{\Sigma P_1^2 L}{EA} = \frac{(1)^2(3)}{16 \times 10^3} + \frac{(1)^2(4)}{16 \times 10^3} = 0.438 \times 10^{-3}$$

WHEN WE HAVE MANY MORE MEMBERS THE TABULAR FORM IS BETTER AS ERRORS ARE LESS LIKELY.

- $\dfrac{\Sigma \int M_1 M_0 \, ds}{EI}$

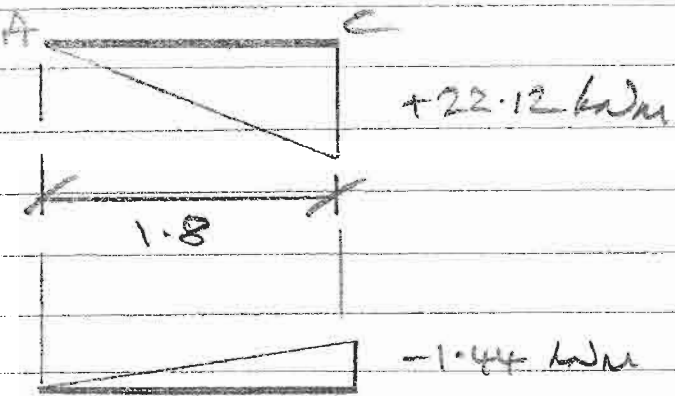
TO DETERMINE THE VIRTUAL WORK DONE BY A MOMENT, WE USE THE INTEGRAL TABLES.

IGNORE MEMBER CD AS M_0 IS ZERO IN THIS MEMBER

ALSO, SAGGING MOMENT IS POSITIVE AND A HOGGING MOMENT IS NEGATIVE.

• LENGTH AC :

M_0 :



M_1 :

$$\int M_1 M_0 \, ds = \frac{1}{3} j k l$$

j IS M_0 MOMENT, k IS M_1 MOMENT

$$\Rightarrow \int \left(\frac{M_1 M_0 \, ds}{EI}\right)_{AC} = \frac{1}{EI}\left[\frac{1}{3}(-1.44)(22.12)(1.8)\right]$$

$$= \frac{-19.11}{EI_{AC}}$$

$$= -2.388 \times 10^{-3}$$

• LENGTH CB :

WE WILL SPLIT THE M_0 DIAGRAM FOR MEMBER CB UP INTO CONSTITUANT PARTS THAT ARE GIVEN IN THE INTEGRAL TABLES.

$\cdot \dfrac{\Sigma M_1^2 ds}{EI}$

WE INTEGRATE THE M_1 DIAGRAM AGAINST ITSELF, USING THE INTEGRAL TABLES:

TRIANGLE'S : $\frac{1}{3} jkl$.

- LENGTH CD :

$$\frac{1}{EI}\left[\frac{1}{3}(0.48)(0.48)(2.4)\right] = \frac{0.184}{EI}$$

- LENGTH AC :

$$\frac{1}{EI}\left[\frac{1}{3}(-1.44)(-1.44)(1.8)\right] = \frac{1.244}{EI}$$

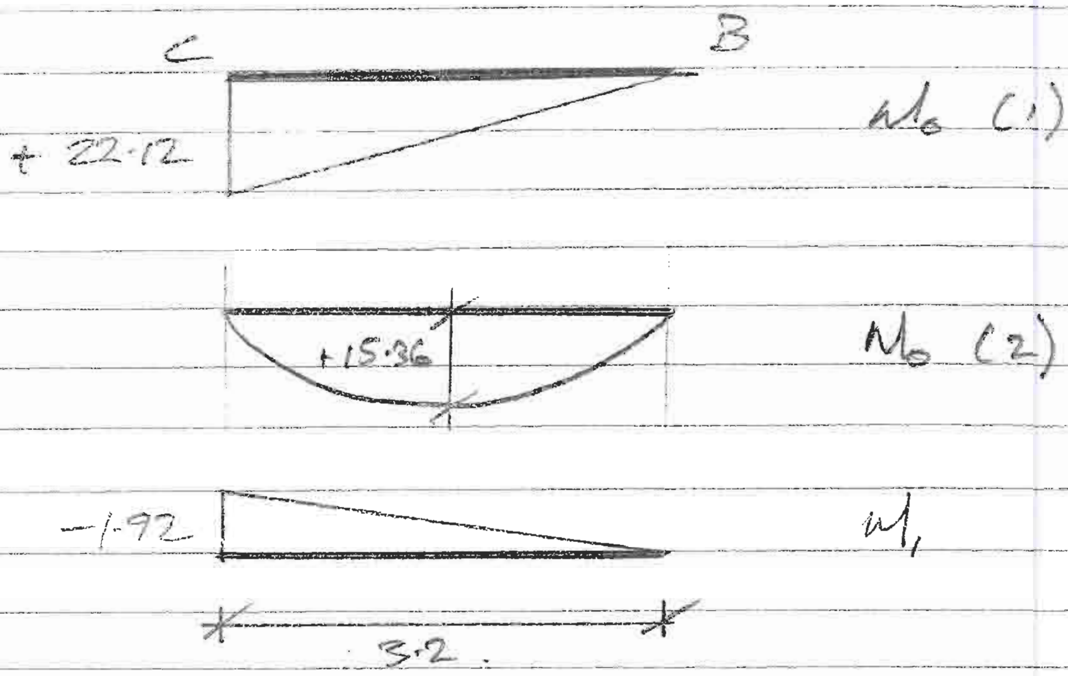
- LENGTH CB :

$$\frac{1}{EI}\left[\frac{1}{3}(-1.92)(-1.92)(3.2)\right] = \frac{3.932}{EI}$$

EI FOR ACB = 8×10^3 kNm^2
EI FOR CD = 4×10^3 kNm^2

$$\Rightarrow \frac{1}{8\times10^3}\left[1.244 + 3.932\right] + \frac{1}{4\times10^3}\left[0.184\right]$$

$$= 0.693 \times 10^{-3}$$



(1) $$\int \frac{M_1 M_o(1)\, ds}{EI} = \frac{1}{EI} \cdot \frac{1}{3}\, j\, k\, l$$

$$\Rightarrow \int \left(\frac{M_1 M_o\, ds}{EI} \right)_{CB(1)} = \frac{1}{EI}\left[\frac{1}{3}(-1.92)(22.12)(3.2) \right]$$

$$= \frac{-45.30}{EI}$$

(2) $$\int \frac{M_1 M_o(2)\, ds}{EI} = \frac{1}{EI} \cdot \frac{1}{3}\, j\, k\, l$$

$$= \frac{1}{EI}\left[\frac{1}{3} \cdot (-1.92)(15.36)(3.2) \right]$$

$$= \frac{-31.46}{EI}$$

$$\Rightarrow \int \frac{M_1 M_o\, ds}{EI} = \frac{-45.30}{EI} + \frac{-31.46}{EI} = \frac{-76.76}{EI} = -9.6 \times 10^{-3}$$

NOW ADD AC & CB $\Rightarrow -11.99 \times 10^{-3}$

NOW, RETURNING TO THE J. W. EQN :

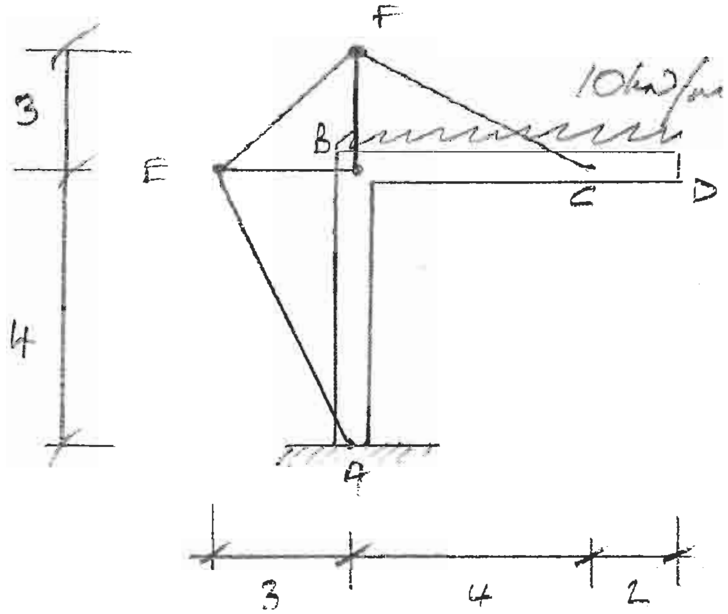
$$0 = 0 + 0.438 \times 10^{-3} x_1 + (-11.99 \times 10^{-3}) + 0.693 \times 10^{-3} x_1$$

$$\Rightarrow 1.131 x_1 = 11.99$$

$$\Rightarrow x_1 = 10.60 \text{ kN}$$

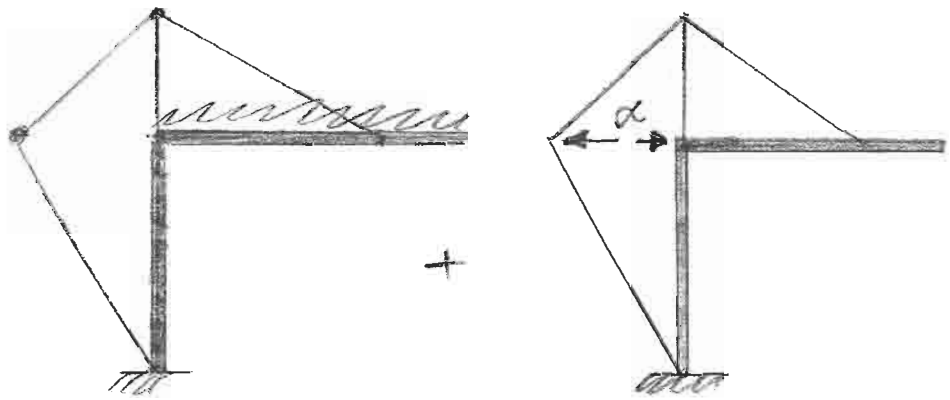
POSITIVE VALUE INDICATES THAT DIRECTION OF x_1 CHOSEN WAS CORRECT
=) CABLE IS IN TENSION AS WE SUSPECTED!

Q2 S'97



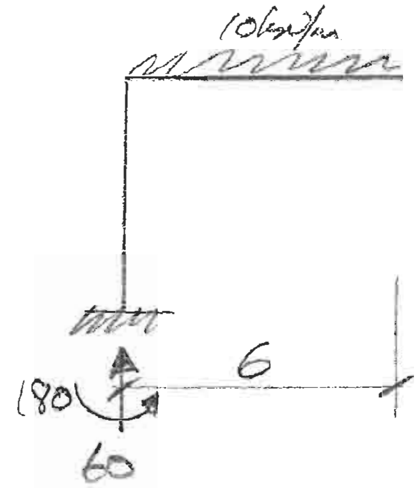
- ABCD:
  $E = 10 \text{ kN/mm}^2$
  $A = 12 \times 10^4 \text{ mm}^2$
  $I = 36 \times 10^8 \text{ mm}^4$
- AEBFC:
  $E = 200 \text{ kN/mm}^2$
  $A = 2 \times 10^3 \text{ mm}^2$

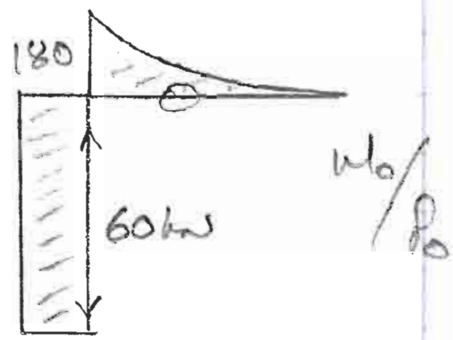
CHOOSE BE AS REDUNDANT:



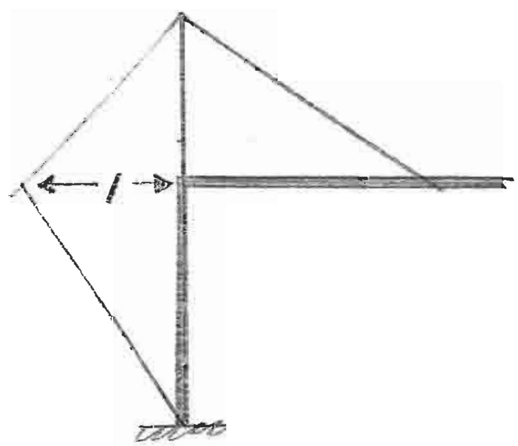
$$P\ M = P_0\ M_0 + \alpha (P_1\ \ M_1)$$

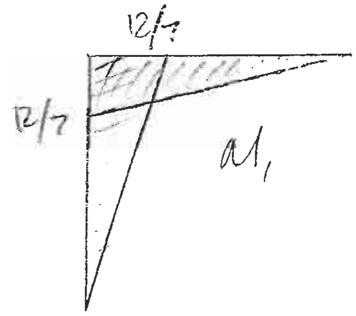
FIRST ANALYSE THE CUT BACK STRUCTURE FOR P_0, M_0.

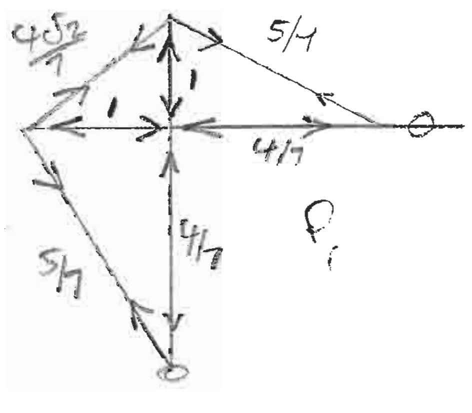


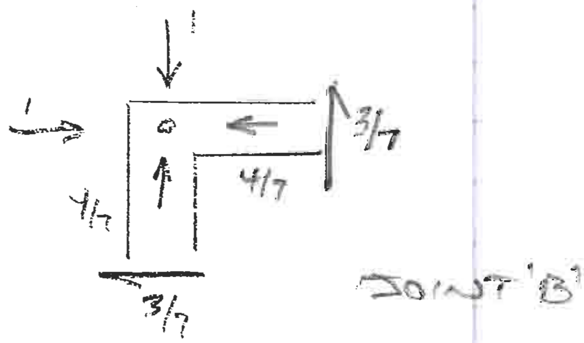


$M = 10 \times 6^2/2 = 180$ kNm $\quad V_A = 10 \times 6 = 60$ kN.









EXT V.W = INT. V.W

EXT VIRT. FORCE × EXT. REAL DISP = INT. V. FORCE × INT. REAL DISP.

$$1 \times 0 = \Sigma \frac{P_1 P L}{AE} + \Sigma \frac{M_1 M\, ds}{EI}$$

$$\Rightarrow \Sigma \frac{P_1 P_0 L}{EA} + x \frac{\Sigma P_1^2 L}{EA} + \int \frac{M_1 M_0 ds}{EI} + x \int \frac{M_1^2 ds}{EI} = 0$$

- $\Sigma \frac{P_1 P_0 L}{EA}$; $\Sigma \frac{P_1^2 L}{EA}$

MEMBER	P_1	P_0	EA	L	$\frac{P_1 P_0 L}{EA}$ $(\times 10^{-6})$	$\frac{P_1^2 L}{EA}$ $(\times 10^{-6})$
AB	$-4/7$	-60	1.2×10^6	4	114.3	1.088
BC	$-4/7$	0	1.2×10^6	4	0	1.088
BE	-1	0	0.4×10^6	3	0	7.5
BF	-1	0	"	3	0	7.5
AE	$5/7$	0	"	5	0	6.377
EF	$\frac{4\sqrt{2}}{7}$	0	"	$3\sqrt{2}$	0	6.927
FC	$5/7$	0	"	5	0	6.377
CD	0	0	1.2×10^6	2	0	0
				$\Sigma =$	114.3	36.86

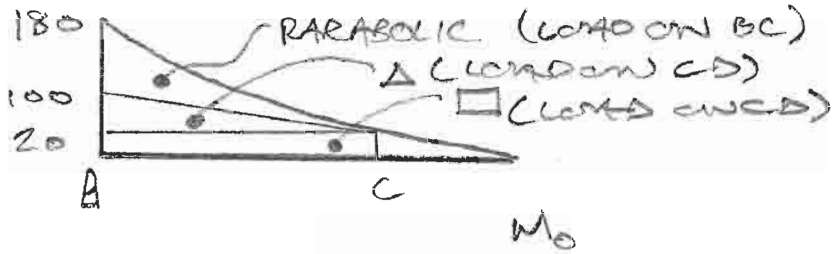
- ABCD: $EA = (10)(12\times10^4)$ kN $= 1.2\times10^6$ kN
- AEBFC: $EA = (200)(2\times10^3)$ kN $= 0.4\times10^6$ kN.

• $\int \frac{M_1 M_0 \, ds}{EI}$:

$$EI = (10)(36 \times 10^8)\,kN\,mm^2 / (10^3)^2 (mm^2/m^2)$$
$$= 36 \times 10^3\,kN\,m^2$$

• AB = $\frac{1}{EI}\left[\frac{1}{2}\left(-\frac{12}{7}\right)(180)(4)\right] = \frac{-617}{EI} = \underline{\underline{-17142.8 \times 10^{-6}}}$

• BC



• PARABOLIC :
$$1/EI\left[\frac{1}{4}\left(-\frac{12}{7}\right)(80)(4)\right] = \frac{-137.14}{EI}$$

• TRIANGLE :
$$1/EI\left[\frac{1}{3}\left(-\frac{12}{7}\right)(80)(4)\right] = \frac{-182.86}{EI}$$

• RECTANGLE :
$$1/EI\left[\frac{1}{2}\left(-\frac{12}{7}\right)(20)(4)\right] = \frac{-68.57}{EI}$$

$\Rightarrow$ BC $= \frac{1}{EI}(-137.14 - 182.86 - 68.57) = \underline{\underline{-10793.61 \times 10^{-6}}}$

$\Rightarrow \int \frac{M_1 M_0 \, ds}{EI} = \underline{\underline{-27,936.41 \times 10^{-6}}}$

- $\int \frac{M_1^2 ds}{EI}$:

$$= \frac{2}{EI}\left[\frac{1}{3}\left(\frac{-12}{7}\right)^2(4)\right] = \underline{\underline{217.69 \times 10^{-6}}}$$

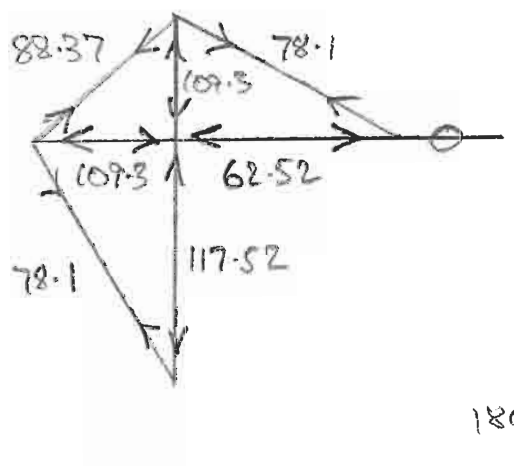
FILL VALUES INTO EQN.:

$$114.3 + 36.86\alpha - 27{,}936.41 + 217.69\alpha = 0$$

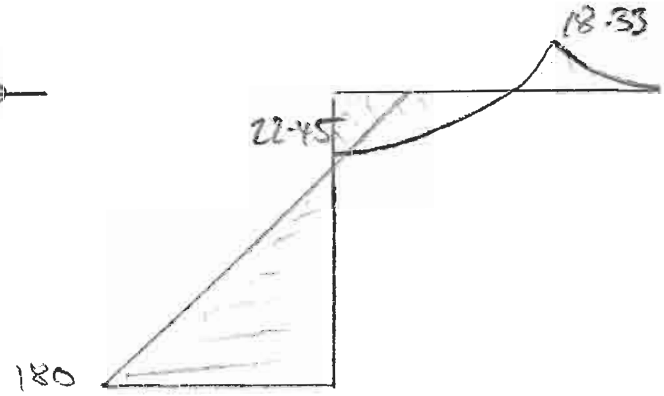
$$\Rightarrow \underline{\underline{\alpha = 109.3}}$$

FINAL VALUES:

$$P = P_0 + \alpha P_1$$
$$M = M_0 + \alpha M_1$$

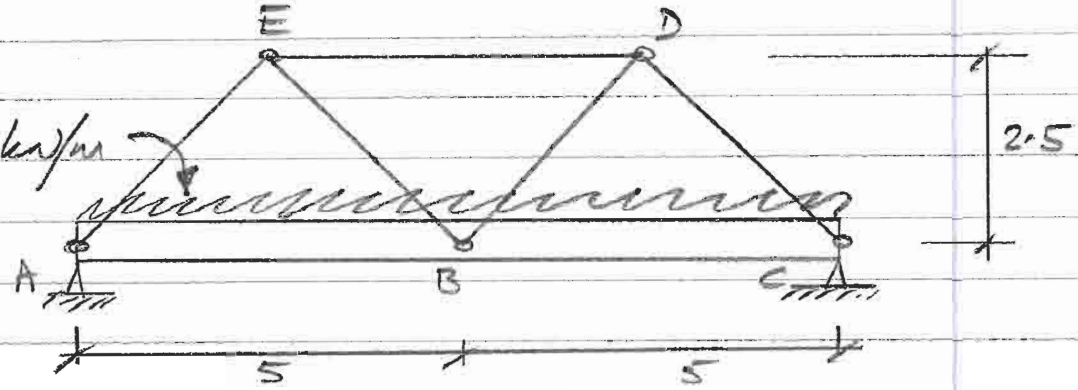


P



M

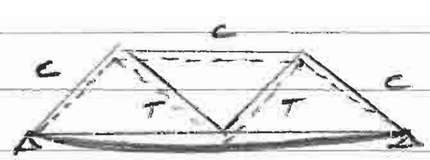
EXAMPLE 3



PROBLEM: CALCULATE THE MAX INTENSITY OF UDL, w, THAT CAN BE CARRIED BY THE BRACED SPAN ABC. THE ALLOWABLE BENDING STRESS IS ≯ 150 N/mm² AND THE ALLOWABLE DIRECT AXIAL STRESS (T OR C) IS ≯ 100 N/mm². THE BEAM ABC IS 500 mm DEEP, ITS AREA IS 6×10^4 mm² AND ITS $I = 125 \times 10^7$ mm⁴. ALL OTHER MEMBERS: A = 1000 mm². ALSO, E IS CONSTANT THROUGHOUT.

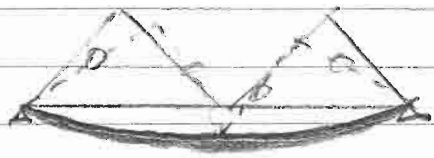
REDUNDANCY:

'IMAGINE' DSD OF THE ABOVE:



'TRUSS' ACTION

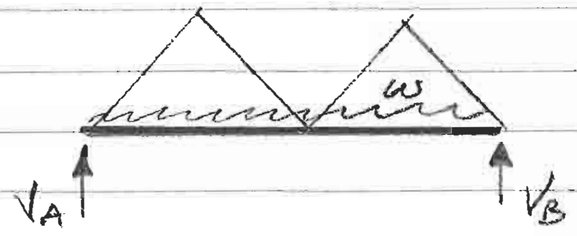
IF WE REMOVE DE:



'BEAM' ACTION.

IT IS APPARENT THAT IT IS A 1°
REDUNDANT STRUCTURE. CHOOSE
DE AS OUR REDUNDANT.

P_o, M_o

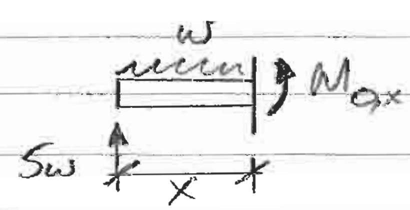


NOTE: IT IS CLEAR FROM THE PROBLEM
THAT WE NEED, SOMEHOW, TO FIND
AN EXPRESSION FOR THE STRESSES
IN THE MEMBERS, IN TERMS OF w.
ONLY THEN CAN WE PUT IN OUR
MAX ALLOWABLE STRESS AND SOLVE
FOR w. START BY ESTABLISHING
P_o, M_o ACTIONS IN TERMS OF w!

$\Sigma F_y = 0 \Rightarrow V_A = V_B = 5w$ (kN)

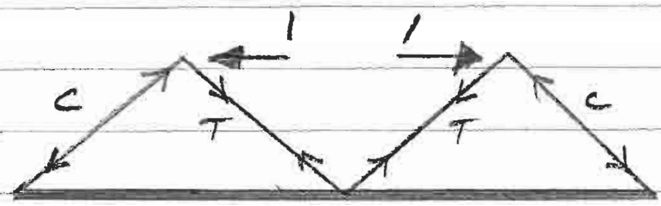
$P_o = 0$ ALL MEMBERS.

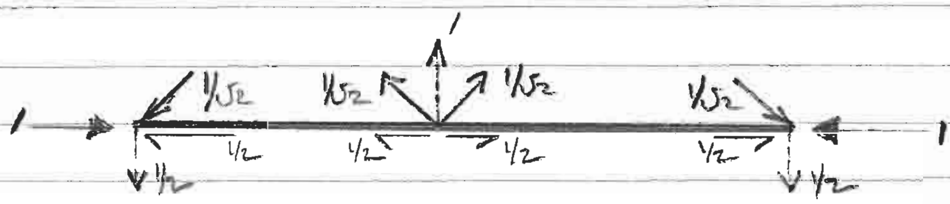
$M_o =$

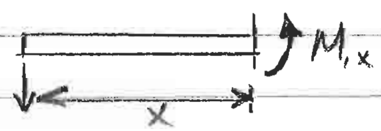


$$M_{ox} = 5wx - wx^2/2$$

P_1, M_1







NOTE: IF THE BEAM SUPPORTS WERE A PIN & ROLLER THEN WE WOULD HAVE AN AXIAL STRESS IN THE BEAM AS WELL AS THE BENDING STRESSES. HOWEVER, WE HAVE A PIN-PIN SUPPORT SYSTEM AND THE SUPPORTS THEREFORE APPLY RESTRAINT AND NO AXIAL FORCES ARE PRESENT (THE AXIAL STRESS DUE TO THE RESTRAINED LONGITUDINAL DISPLACEMENT IS CONSIDERED NEGLIGABLE). ALSO, NOTICE THERE ARE NO VERTICAL REACTIONS: $\Sigma F_y = 0$.

$$M_{1x} = -\tfrac{1}{2}x$$

- M ⊕ = TENSION IN BTM OF BEAM
- P ⊕ = AXIAL TENSION.

VIRTUAL WORK EQUATIONS:

Σ EXT. WORK = $1 \times 0 = 0$

Σ INT. WORK = $\Sigma P_1 \rho + \Sigma M_1 m$

THEREFORE THE FOLLOWING APPLIES:

$$\alpha_1 = -\left[\frac{\int_0^L \frac{P_1 P_0 dx}{EA} + \int_0^L \frac{M_1 M_0 ds}{EI}}{\int_0^L \frac{P_1^2 dx}{EA} + \int_0^L \frac{M_1^2 ds}{EI}}\right]$$

IN ORDER TO EVALUATE THE VALUE OF EACH OF THE ABOVE TERMS A TABLE WOULD BE BENEFICIAL:

MEMBER	EA	EI	M_0	P_0	M_1	P_1	LIMITS
AB	6×10^{-2}	1.25×10^{-3}	$5wx - wx^2/2$	0	$-x/2$	0	$0 \to 5$
BC	6×10^{-2}	1.25×10^{-3}	$5wx - wx^2/2$	0	$-x/2$	0	$0 \to 5$
AE	1×10^{-3}	–	–	0	0	$-1/\sqrt{2}$	$0 \to 2.5\sqrt{2}$
DB	1×10^{-3}	–	–	0	0	$+1/\sqrt{2}$	$0 \to 2.5\sqrt{2}$
BE	1×10^{-3}	–	–	0	0	$+1/\sqrt{2}$	$0 \to 2.5\sqrt{2}$
DC	1×10^{-3}	–	–	0	0	$-1/\sqrt{2}$	$0 \to 2.5\sqrt{2}$
DE	1×10^{-3}	–	–	0	0	-1	$0 \to 5$

NOW WE CAN CONTINUE AND EVALUATE THE INTEGRALS.

- $\int_0^L \frac{P_1 P_0 \, dx}{EA} = \underline{0}$ AS $P_0 = 0$ FOR ALL.

- $\int_0^L \frac{M_1 M_0 \, dx}{EI} = 2\int_0^5 \left[(5wx - wx^2/2)(-x/2) / 1.25 \times 10^{-3} \right] dx.$

$$= \frac{2\times10^3}{1.25}\int_0^5 \left(\frac{wx^3}{4} - \frac{5wx^2}{2}\right) dx$$

$$= \frac{2\times10^3}{1.25}\left[\frac{wx^4}{16} - \frac{5wx^3}{6}\right]_0^5$$

$$= \frac{2\times10^3 w}{1.25}\left[\frac{5^4}{16} - \frac{5\times5^3}{6}\right]$$

$$\underline{= -104{,}167\,w}$$

- $\int_0^L \frac{P_1^2 \, dx}{EA}$

  - MEMBERS AE, DB, BE, DC : - MEMBER DE :

$$= \frac{4}{EA}\int_0^{2.5\sqrt{2}} (\pm 1/\sqrt{2})^2 \, dx + \frac{1}{EA}\int_0^5 (1)^2 \, dx$$

NOTE : EA CONSTANT, $(\pm 1/\sqrt{2})^2 = +1/2$

$$= \frac{4\times10^3}{2}\left[x\right]_0^{2.5\sqrt{2}} + 1\times10^3\left[x\right]_0^5$$

$$= 7071 + 5000 = \underline{12071}$$

$$\int_0^L \frac{M_1^2 ds}{EI} = \frac{2\times10^3}{1.25}\int_0^5 (-x/2)^2\, dx$$

$$= \frac{2\times10^3}{1.25}\int_0^5 \frac{x^2}{4}\cdot dx$$

$$= \frac{2\times10^3}{1.25}\left[x^3/12\right]_0^5$$

$$= \frac{2\times10^3}{1.25}\cdot\frac{5^3}{12} = \underline{16,667}$$

<u>EVALUATE:</u>

$$\alpha_1 = -\left[\frac{-104,167\,w}{12,071 + 16,667}\right] = +3.625w.$$

THE POSITIVE VALUE INDICATES THAT THE DIRECTION ASSUMED FOR THE UNIT LOAD WAS CORRECT.

<u>AXIAL LOADS</u>

$$P = P_0 + \alpha_1 P_1$$

AE & CD: $P = 0 + 3.625w(-1/\sqrt{2}) = -2.563w.$ (C)

DB & BE: $P = 0 + 3.625w(+1/\sqrt{2}) = +2.563w$ (T)

DE: $P = 0 + 3.625w(-1) = -3.625w$ (C)

THUS OUR IMAGINARY DSD WAS CORRECT IN ITS ASSIGNMENT OF TENSION OR COMPRESSION FORCES IN THE TRUSS.

SHEAR FORCES

WE ARE NOT REQUIRED TO ESTABLISH THESE IN THE PROBLEM.

BENDING MOMENTS

$$M = M_0 + \alpha_1 M_1$$

WE ARE ONLY CONCERNED WITH ABC:

$$M_{ABC} = \left\{ 5wx - wx^2/2 \right\} + \left\{ (3.625w)(-x/2) \right\}$$

$$= 3.1875wx - 0.5wx^2$$

Maximum Moment occurs at $\frac{dM}{dx} = 0$.

$$\Rightarrow \frac{dM}{dx} = 3.1875w - wx = 0$$

$$\Rightarrow \quad x = 3.1875\,m$$

$$\Rightarrow M_{max} = (3.1875)^2 w - \frac{(3.1875)^2 w}{2}$$

$$= 5.08w \quad (kNm)$$

DETERMINE w_{MAX}

TWO ACTIONS & ASSOCIATED STRESSES TO CHECK:

- AXIAL
- BENDING.

• AXIAL : MAX $\sigma = 100\ N/mm^2$

$$\sigma_{max} = \frac{F_{max}}{A}$$

$$100 = \frac{3.625\omega \times 10^3}{1000} \quad \frac{(N)}{(mm^2)}$$

$$\Rightarrow \omega = \frac{1 \times 10^5}{3.625 \times 10^3}$$

$$= \underline{27.59\ kN/m} \quad (N/mm = kN/m)$$

• FLEXURAL : MAX $\sigma = 150\ N/mm^2$

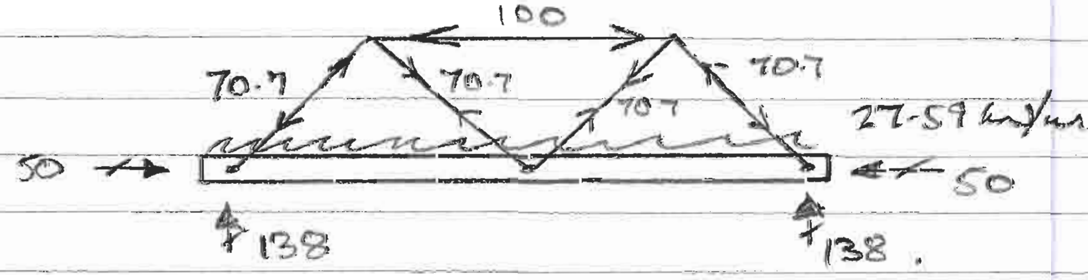
$$\sigma_{max} = \frac{My}{I}$$

$$150 = \frac{5.08\omega \times 10^6 \times (500/2)}{125 \times 10^7}$$

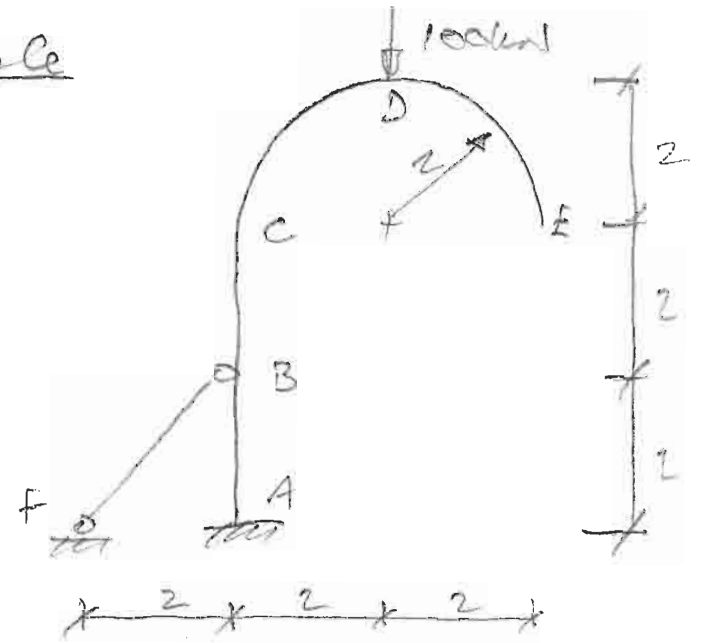
$$\Rightarrow \omega = 150 \times 0.984$$

$$= \underline{147.64\ kN/m}$$

THUS THE MAXIMUM UDL IS DETERMINED BY THE AXIAL COMPRESSIVE FORCE IN MEMBER DE.



Example



$EI = 120 \times 10^3\ kNm^2$

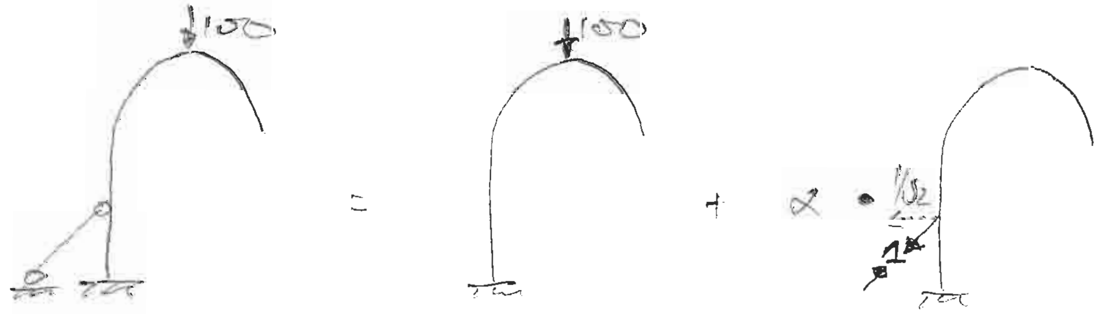
$EA = 60 \times 10^3\ kN$

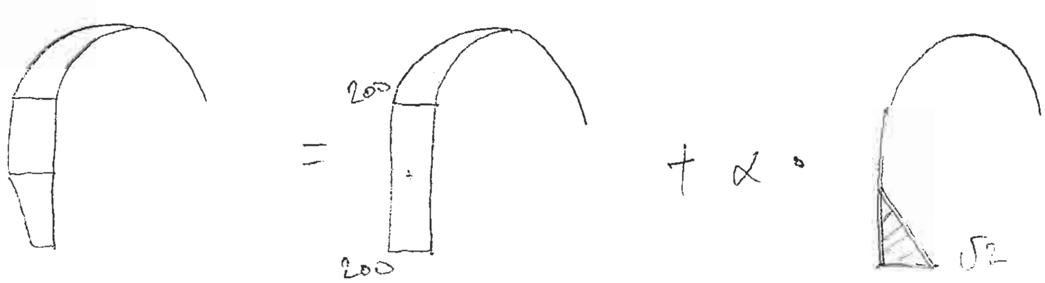
For this structure:

(a) draw the bending moment diagram;

(b) find δ_{Ey}

(a) Choosing BF as the redundant gives:





$M \qquad M_0 \qquad M_1$

For virtual work:

$$\delta W = 0$$

$$\delta W_E = \delta W_I$$

$$y \cdot \delta F = \sum e \cdot \delta P + \sum \theta \cdot \delta M$$

$$0 \cdot 1 = \sum \frac{PL}{EA} \cdot \delta P + \sum \int_0^L \frac{M}{EI} \cdot \delta M \, dx$$

But $P = P_0 + \alpha P_1$ & $M = M_0 + \alpha M_1$

$$\therefore 0 = \sum \frac{(P_0 + \alpha P_1) L \delta P}{EA} + \sum \int_0^L \frac{(M_0 + \alpha M_1)}{EI} \cdot \delta M \cdot dx$$

But P_1 & δP are equivalent, as are M_1 & δM

$$\therefore 0 = \sum \frac{P_1 P_0 L}{EA} + \alpha \cdot \sum \frac{P_1^2 L}{EA} + \sum \int_0^L \frac{M_0 M_1}{EI} \cdot dx + \alpha \cdot \sum \int_0^L \frac{M_1^2}{EI} \cdot dx$$

Calculating each term separately:

- $\sum \frac{P_1 P_0 L}{EA} = 0$ since $P_0 = 0$
- $\sum \frac{P_1^2 L}{EA} = \frac{(1)^2 \cdot 2\sqrt{2}}{EA} = \frac{2\sqrt{2}}{EA}$ for member BF
- $\int_A^B \frac{M_1 M_0 \, dx}{EI} = \frac{1}{EI}\left[\frac{1}{2}(200)(-\sqrt{2})(2)\right] = \frac{-200\sqrt{2}}{EI}$
- $\int_A^B \frac{M_1^2 \, dx}{EI} = \frac{1}{EI}\left[\frac{1}{3} \cdot (-\sqrt{2})(-\sqrt{2})(2)\right] = \frac{4/3}{EI}$

Thus:

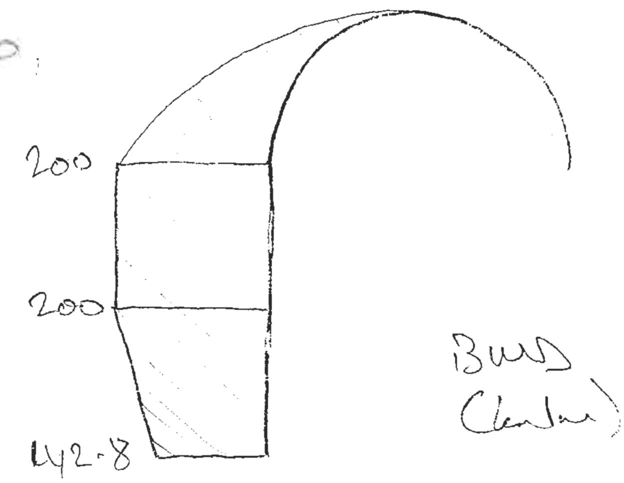
$$0 = 0 + \alpha \cdot \frac{2\sqrt{2}}{60\times10^3} - \frac{200\sqrt{2}}{120\times10^3} + \alpha \cdot \frac{4/3}{120\times10^3}$$

$$\therefore\ 0 = \alpha\left(4\sqrt{2} + 4/3\right) - 200\sqrt{2}$$

$$\therefore\ \alpha = \frac{200\sqrt{2}}{4\sqrt{2} + 4/3} = +40.46$$

Thus the force in BF is 40.46 kN in tension.

Also,

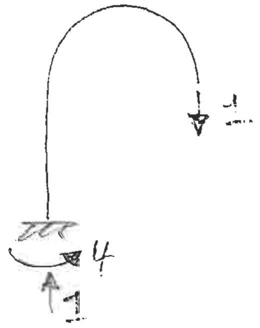


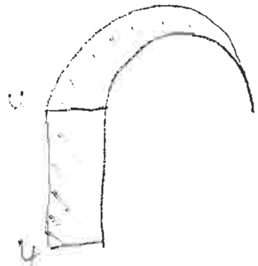
$M = M_0 + \alpha M_1$

$\therefore M_A = 200 + 40.46(-\sqrt{2})$

$= 142.8$ kNm

(3) To find δ_{Ey}, we apply a unit load at E vertically downwards. We need only use the primary structure however, since it forms an equilibrium set:



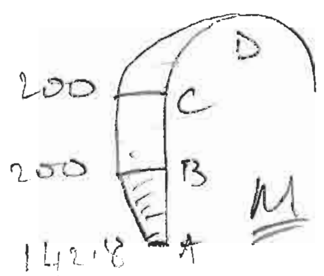


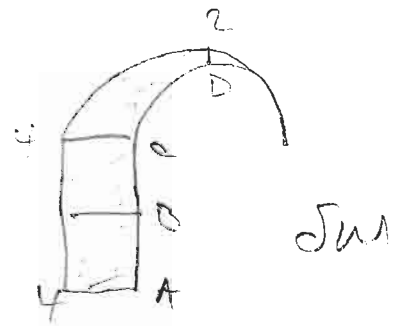
δM

For a deflection:

$$\delta_{EV} \cdot 1 = \sum \frac{PL}{EA} \cdot \delta P + \sum \int_0^L \frac{M}{EI} \cdot \delta M \cdot dx$$

Since $\delta P = 0$ we need only calculate the second term:





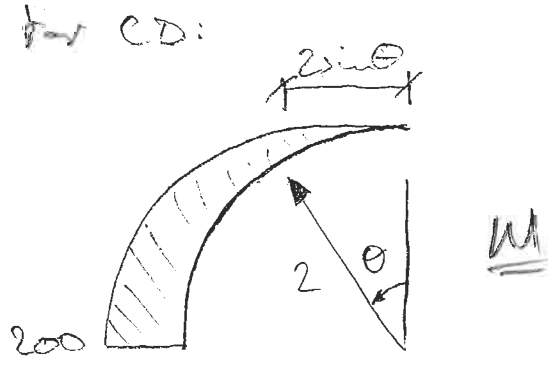
- for AB:

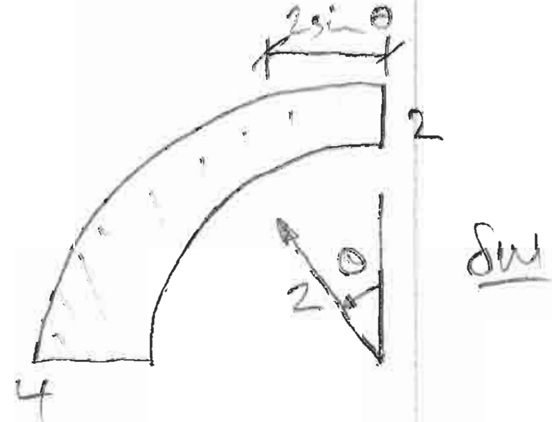
$$\int \frac{M \cdot \delta M}{EI} = \frac{1}{EI}\left[\frac{1}{2}(200 + 142.8)(4)(2)\right] = \frac{1371.2}{EI}$$

- for BC:

$$\int \frac{M \cdot \delta M}{EI} = \frac{1}{EI}\left[(200)(4)(2)\right] = \frac{1600}{EI}$$

- for CD:

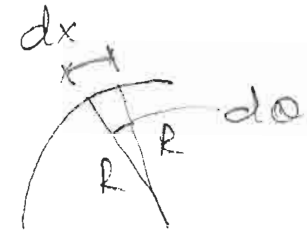




$$M(\theta) = 100 \times 2\sin\theta$$
$$= 200\sin\theta$$

$$\delta M(\theta) = 2 + 1 \times 2\sin\theta$$
$$= 2 + 2\sin\theta$$

Also, note that since we are now integrating using the angle, we must change the dx to a $d\theta$:



$$\therefore dx = R \cdot d\theta = 2d\theta$$

Thus: $$\int_C^D \frac{M \cdot \delta m}{EI} dx = \int_0^{\pi/2} (200\sin\theta)(2+2\sin\theta) \cdot 2\,d\theta$$

$$= \int_0^{\pi/2} (800\sin\theta + 800\sin^2\theta)\, d\theta$$

$$= 800\int_0^{\pi/2} \sin\theta\, d\theta + 800\int_0^{\pi/2} \sin^2\theta \cdot d\theta$$

Taking each term in turn:

$$\int_0^{\pi/2} \sin\theta\, d\theta = -\cos\theta \Big|_0^{\pi/2} = -0 - (-1) = +1$$

$$\int_0^{\pi/2} \sin^2\theta\, d\theta = \left[\frac{\theta}{2} - \frac{1}{4}\sin^2\theta\right]_0^{\pi/2}$$

$$= \left(\frac{\pi}{4} - \frac{1}{4}(1)^2\right) - \left(0 - \frac{1}{4}(0)^2\right)$$

$$= \pi/4 - 1/4$$

$$\therefore \int_C^D \frac{M \cdot \delta m}{EI} \cdot dx = 800(1) + 800\left(\frac{\pi}{4} - 1/4\right) = \underline{\frac{200\pi + 600}{EI}}$$

Thus:

$$\delta_{Ey} = \frac{1371.2}{EI} + \frac{1600}{EI} + \frac{200\pi + 600}{EI}$$

$$= +\frac{4199.5}{EI}$$

For $EI = 120 \times 10^3 \text{ kNm}^2$, we have:

$$\delta_{Ey} = +35\text{mm} \downarrow$$